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COMPARISON OF POST-STORM NON-ADIABATIC RECOVERY OF TRAPPED PROTONS WITH RADIAL DIFFUSION

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ABSTRACT

During magnetic storms it has been observed that the geomagnetically trapped protons undergo rapid non-adiabatic changes followed by slow non-adiabatic recovery approaching the pre-storm values. The slow non-adiabatic recovery can be accounted for in a semiquantitative way by solving a time dependent Fokker-Planck equation with radial transport and loss terms describing coulomb energy degradation and charge exchange. The equation is solved numerically in a region of space where we have measurements of the 100 keV to 1700keV protons mirroring at the equator. The transport term is assumed to have the form $D = k\mu^m L^p$ where μ is the magnetic moment of the proton and L the McIlwain shell parameter. The value of D which gives the best fit to the data is found. Due to the limited amount of data used in this study the μ and L dependence of the diffusion coefficient are not determined very accurately. The resulting values of D was found to be larger than the value evaluated by Nakada and Mead. The e-folding time for the intensities of the higher energies to recover at L = 3.5 is of the order of a year.

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INTRODUCTION

A short time after the discovery of the geomagnetically trapped particles it was suggested by Kellogg (1959) that the spatial and temporal behavior of these particles might be influenced by processes that produced departure from first order adiabatic motion, resulting in diffusion across magnetic shells, conserving the first two adiabatic invariants and violating the flux invariant. Such a process has been effectively employed by Nakada and Mead (1965) in an attempt to arrive at an equilibrium outer zone proton distribution.

An apparent radially inward motion of MeV electrons has been observed by Frank (1965) near the equatorial plane and at high latitudes by McDiarmid and Burrows (1966) and by Craven (1966). One possible interpretation of these data invokes trans-L diffusion. Newkirk and Walt (1968) have evaluated the radial diffusion coefficient for electrons at low L-values and Kavanagh (1968) for electrons in the region four to seven earth radii.

Søraas and Davis (1968) have made a study of the temporal behavior of the 100 keV to 1700 keV trapped protons in the range 2 to 5.5 earth-radii for the period 29 of January through 29 of June 1965. They found that the particle intensities exhibit both adiabatic changes which varied directly with the magnetic field (Dst) and non-adiabatic changes which do not track the field. They were able to transform the proton intensities measured in the time variable field to a reference dipole field using a transformation which conserved the three adiabatic invariants of motion. The transformed intensities then supposedly show time variations due only to non-adiabatic processes in which at least one invariant was violated.

Their results show two types of non-adiabatic variations. First during magnetic storms the protons underwent rapid changes. Protons having energies less than about 200 keV were enhanced while higher energy protons were depleted. During the 18 April 1965 storm, which was the largest for this period, the enhancement was as much as a factor of four and the depletion as much as a factor of 10. The second type was a slow post-storm recovery of both high and low energy protons toward their prestorm values. The recovery times were observed to decrease with increasing radial distance. They also observed that the proton energies before the large april 18 storm exhibited the L^{-3} dependence characteristic of cross-L diffusive equilibrium and did not show this after the storm, though they slowly returned toward this dependence. These last two results coupled with the results of Nakada and Mead (1965) suggest that the recover phase might be governed by radial diffusion of the protons.

In the present analysis the temporal behavior of the 100 keV to 1700 keV protons between the April 18 and June 15, 1968 geomagnetic storms will be compared with a calculated time-dependent radial diffusion model and used to evaluate the radial diffusion coefficient.

DIFFUSION CALCULATION

The equation used to describe the motion across L-shells of protons mirroring at the equator is a one-dimensional Fokker-Planck equation with radial transport terms and loss terms describing coulomb energy degradation and charge-exchange.

$$\frac{\delta n}{\delta t} = -\frac{\delta}{\delta r}\left(D_1 n\right) + \frac{1}{2}\frac{\delta^2}{\delta r^2}\left(D_2 n\right) - \frac{\delta}{\delta \mu}\left(\left\langle\frac{\Delta\mu}{\Delta t}\right\rangle n\right) - \frac{n}{\tau} \qquad (1)$$

where $n(r, \mu, J, t)\, dr\, du\, dJ$ is the number of particles between equatorial radius r and $(r + dr)$, having magnetic movement between μ and $(\mu + d\mu)$, and integral invariant J to $(J + dJ)$ at the time t. The coefficient D_1 is the mean radial displacement per unit time $\langle\Delta r/\Delta t\rangle$ and D_2 is the mean square radial displacement per unit time $\langle(\Delta r)^2/\Delta t\rangle$. The angular brackets denote time averages. The third term on the right hand side of the equation describs coulomb energy loss and the fourth term charge exchange loss. τ is the e-folding lifetime for charge exchange. In the case of coulomb energy loss the higher energy particles are a source for low energy particles whereas charge exchange removes particles catastrophically. Any pitch angle scattering of the protons are neglected. The equation is the same as used by Nakada and Mead (1965) and similar to those used by Davis and Chang (1962), Tverskoy (1964), and Newkirk and Walt (1968) in studies of trapped particles.

Fälthammar (1966) has shown that D_1 and D_2 are related by

$$D_1 = \frac{r^2}{2.}\frac{\delta}{\delta r}\left(\frac{D_2}{r^2}\right) = r^2\frac{\delta}{\delta r}\left(\frac{D}{r^2}\right) \tag{2}$$

where

$D = D_2/2$.

As a consequence the Fokker-Planck equation (1) can be rewritten as a diffusion equation of the form

$$\frac{\delta n}{\delta t} = \frac{\delta}{\delta r}\left(\frac{D}{r^2}\frac{\delta}{\delta r}\left(r^2 n\right)\right) - \frac{\delta}{\delta \mu}\left(\left\langle\frac{\Delta\mu}{\Delta t}\right\rangle n\right) - \frac{n}{\tau} \tag{3}$$

Fälthammar (1967) has found a general expression for the diffusion coefficient D which depends on the power spectrum of the electromagnetic disturbances. He points out that if the magnetic disturbances have a power spectrum falling off with the frequency raised to the negative exponent of s then the diffusion coefficient is of the form

$$D_M \propto r^{6+2s} \cdot \mu^{2-s} \tag{4}$$

where the subscript M on D is used to indicate that this form of the diffusion coefficient is due to magnetic disturbances and not to time dependent electric potential fields.

The diffusion coefficient is here assumed to have the form

$$D = k \cdot \mu^m \cdot r^P \tag{5}$$

By choosing different values of k, m and p Equation (3) can be solved for different values of the diffusion coefficient.

For the coulomb energy loss and charge-exchange lifetimes entering into Equation (3) the same expressions as used by Nakada and Mead (1965) are used. They are

$$\left\langle\frac{\Delta\mu}{\Delta t}\right\rangle = -3.55 \cdot 10^{-6}\, \rho \left(\frac{r^6}{\mu}\right)^{1/2} \qquad \text{MeV/Gauss} \cdot \text{day}$$

with r in earth radii, ρ in electrons/cm^3 and μ the magnetic moment of the protons in MeV/Gauss. The electron density is given by

$$\rho = (8000/r^4 + 50) \qquad \text{electrons/cm}^3$$

the charge exchange term is $1/\tau = \sigma\rho_0\, v$, where ρ_0 is the neutral hydrogen density, given by

$$\rho_0 = 7.35 \cdot 10^3/r^5 \qquad \text{atoms/cm}^3$$

For the charge exchange cross section, σ, values given by Allison (1958) were used v is the proton velocity.

Using these expressions together with expression (5) for the diffusion coefficient the transport equation (3) can be written

$$\frac{\delta n}{\delta t} = k\mu^m r^p \left(\frac{\delta^2 n}{\delta t^2} + \frac{2+p}{r}\frac{\delta n}{\delta t} + \frac{2\cdot(p-1)}{r^2}\, n\right)$$

$$+ 3.55 \cdot 10^{-6}\,(8000 \cdot r^{-4} + 50) \cdot \frac{\delta}{\delta\mu}\left(\left(\frac{r^9}{\mu}\right)^{1/2} n\right) - \sigma\rho_0\, vn \qquad (6)$$

The time evolution of the distribution function n can now be calculated by solving Equation (6) with the appropriate initial - and boundary conditions.

In order to obtain the initial distribution, the observed integral proton intensities have to be related to n. Nakada and Mead (1965) have shown that

$$n(r, \mu, J = 0, t) \propto r \cdot j(r, E, t) \tag{7}$$

where $j(r, E, t)$ is the differential flux (protons/cm^2-s-sr-MeV). The integral invariant J is equal to zero for particles mirroring at the equator.

As shown previously by Davis and Williamson (1966) the integral energy spectra measured by the scintillator experiment on board Explorer 26 can quite nicely be fitted with an exponential expression. Thus the differential flux is approximated by

$$j\left(r_0, E, t_0\right) = - N_0/E_0 \cdot \exp\left(-E/E_0\right) . \tag{8}$$

Using this equation and the definition of the magnetic moment of particles mirroring at the equator,

$$\mu = E/B \tag{9}$$

we obtain the following expression for n,

$$n\left(r_0, \mu, J = 0, t_0\right) = CN_0\, r/E_0 \exp\left(-\mu B/E_0\right) . \tag{10}$$

C is a constant of proportionality and the subscript on r and t indicate that n is evaluation at a radial distance r_0 at a particular time t_0.

The left-hand side of Figure 1 shows the measured radial intensity distribution $J(>E, r, t_0)$ of protons mirroring at the equator for the eight energies listed in the figure on day 111 (April 21) of 1965. From this mapping the distribution $n(r, \mu, J = 0, t_0)$ is obtained and shown on the right-hand side of the figure. Quadiatic interpolation is used in the r-direction to obtain n in the region of μ-r space needed to solve Equation (6) numerically. Having obtained the initial distribution one has to choose the boundary conditions in order to solve Equation (6). Due to the increased atmospheric density and the corresponding heavy loss of particles from charge exchange and coulomb energy degradation, one knows and observes that the distribution function n goes to zero close to the earth. Somewhat arbitrarily n is set equal to zero at $r = 1.5$ earth radii. At $r = 5.5$ earth radii the proton intensity is assumed to be independent of time at all energies, as shown by Söraas and Davis (1968) to be approximately true. They show that the intensities at $L = 5.0$ both before and after the April 18 storm on the average run fairly constant, though significant variations occurred on a short time scale. For the time period and space region considered here, the results of the calculations do not depend critically on the boundary conditions as will be shown later.

RESULTS AND DISCUSSIONS

The time dependence of the protons have been calculated by solving Equation (6) numerically on a computer with the initial and boundary values as outlined in the previous section.

Three sets of values for the parameters in the diffusion coefficient expression (5) have been considered;

Set A. $k = 0.31 \times 10^{-9}$, $m = 0.0$, and $p = 10$ as evaluated by Nakada and Mead (1965) using the observed frequency and size of sudden commencements and sudden impulses.

Set B. $m = 0.0$ and $p = 10$ with k varied to minimize the RMS between calculated and measured values.

Set C. All three parameters k, m, and p varied to minimize the RMS.

With Set B and C where the least squares method was used to determine parameter values only data at $L = 3.0$, 3.5, and 4.0 earth radii were used. Data at $L = 4.5$ will be compared with calculated values but were excluded in the fitting process because the calculated values at this L-value are sensitive to the assumed $L = 5.5$ boundary values as is shown below.

Figure 2 shows the calculated and observed integral intensities as a function of time for the four L-values 3.0, 3.5, 4.0, and 4.5 earth radii and for each of eight proton energies 134, 180, 220, 345, 513, 775, 1140, and 1700 keV. The calculated time variation for parameter set A are shown as dashed curves and for set B as solid curves. The measured points are the x's. The prestorm intensities are indicated to show the rapid changes which took place during the storm and that the post-storm intensities slowly return to these values. It should be noted that the measured values and the values calculated from the initial distribution on day 111 are not in complete agreement. This is because the measured spectra are not exactly exponential as assumed.

Comparison between the calculated curves and measured points in Figure 2 shows that the time variation computed with parameter set A does not follow the observed variation whereas those computed with parameter set B are reasonably good fits to the data. Thus the diffusion coefficient evaluated by Nakada and Mead (1965) is too small by a factor of 8 to account for the observed time variation. This agrees with their further results that to get good agreement between the steady-state proton flux measured in 1962 and their calculated distribution, their diffusion coefficient needed to be increased by a factor of 8. Thus approximately the same value of the diffusion coefficient is needed in order to bring the diffusion model in agreement with the steady state proton distribution measured in 1962 and with the time behavior of the protons after the April 18, 1965 geomagnetic storm. This leads to the conclusion that the field fluctuations driving the diffusion had approximately the same intensity level during these two time periods. It also rather conclusively confirms that L-diffusion plays a major role in populating the outer zone protons. Of course other non-adiabatic processes, such as that producing the rapid storm time changes observed by Söraas and Davis (1968), are also important. The time variations calculated for set C are almost identical to the solid curves for set B in Figure 2. The diffusion coefficient parameters obtained for set B and C are listed in Table 1 along with the RMS of the fit measured in decibels. The units are such that D in Equation (5) is given in (earth radii)^2 per day when μ is in MeV/Gauss and r in earth radii. The standard deviations of the parameters listed are based on the statistics of the fit. The value of m in set C came out slightly negative but not significantly different from zero. To determine how sensitive the RMS value is on the value of p the best fit to the experimental results was calculated, for a range of p values assuming $n = 0$. The RMS values exhibit a broad minimum for p values in the range 10 to 13. Due to the high correlation between the parameters k and p, possibly due to the limited and coarse coverage of data in the L-direction, it is not possible to determine p more accurately. The value of the diffusion coefficient D_2 in the L-range 3.0 to 4.0 is, however, fairly accurately determined.

We notice that in Figure 2 the solid lines overlay the dashed lines for the three lowest energies at all L-values. This means that in this model the time behavior of these low energy protons are mainly governed by the loss mechanisms and not by radial diffusion.

The low energy protons at L-values above $L = 3.5$ are observed to decay faster during the time immediately following the storm time enhancement, than the model used here can account for. Losses due to charge exchange, which affect the low energy protons most, are expected to be of less importance at high L-values due to the reduction of the neutral hydrogen density with increasing radial distance from the earth. From this one should expect the decay rate of the low energy protons to go down with increasing L-value. It is thus believed that some other mechanism together with the ones considered here must control the decay rate of the low energy protons. Kennel and Petschek (1966) have shown that pitch angle scattering by ion cyclotron noise sets an upper limit on protons which may be stably trapped. They have further shown that in the region about $L = 4$ the protons are near this limit. It is thus possible that the low energy protons exceeded this limit during their rapid storm-time enhancement. According to Kennel and Petschek (1966) the proton intensity should then be rapidly forced back to their stable trapping limit through a non-linear wave-particle interaction process. This may then explain the fast decay of the low energy protons at high L-values following the storm-time enhancement.

Another way of presenting the data is shown in Figure 3. The distribution function n is plotted vs. radial distance for different values of μ the magnetic moment of the protons and for three different times, 0, 20 and 36 days after the storm. In the left-hand side of the figure n is plotted as obtained from the measured fluxes on Explorer 26. We can see how the shape of these curves changes as time progresses. The changes in n are more rapid at high L-values than at low. In this representation of the observed data the diffusive character of the storm time recovery becomes more apparent than in the plots of integral intensity vs. time. The right-hand side of the figure shows the calculated changes in n as computed from the diffusion equation using $D_2 = 2.40 \cdot 10^{-9} r^{10}$ (earth radii)^2/day. It is seen that the computed time evolution of the distribution function n in general follows the observed one quite closely.

To test how sensitive the diffusion calculations are on the boundary conditions at $L = 5.5$, Equation (6) was solved for different boundary conditions at this L-value using the diffusion coefficient $D_2 = 2.4 \cdot 10^{-9} r^{10}$ (earth radii)^2/day. The value of the distribution function n at $L = 5.5$ was reduced to half of the value used previously. In a time period of 45 days this change in the boundary conditions had no effects at $L = 3.0$ and 3.5. At $L = 4.0$ the calculation started to deviate after 20 days from the one presented in Figure 2. But still after a time of 45 days there was less than one db difference between the two calculations. At $L = 4.5$ the two solutions started to deviate after 4 days and after 45 days the difference was about 2 db. As an extreme the distribution function n was set equally to zero at $L = 5.5$. At $L = 3.0$, 3.5 and 4.0 the results of this calculation was essentially the same as in the case where n was reduced to half its value. At $L = 4.5$ though this last calculation showed a steady drop in the intensities after a 4 day period due to the presence of a sink one earth radii away. As the actual variations in the fluxes at 5.5 earth radii for most of the time are less than 50% and the variations take place on a time scale of a few days, the results of the diffusion calculations do not for the time period and space-region considered in this study, depend critically on that boundary condition.

It is interesting to use the diffusion coefficient $D_2 = 2.40 \cdot 10^{-9} r^{10}$ (earth radii)^2/day to calculate how the distribution function for the protons changes with time during a longer period using the same initial and boundary conditions as before. In Figure 4 the result of this calculation is shown. The distribution function n is plotted vs. radial distance for two values of μ the magnetic moment of the protons. We can see how the shape of these curves changes as time progresses. The protons diffuse inwards approaching the steady state first at large radial distances and later closer to the earth. At first the changes are fairly rapid, but then they slow down as the spatial gradient in the distribution function decreases. The time for the high energy protons which were depleted during the storm to complete 62% of the recovery to prestorm value are approximately 380 days at L = 3.5, 240 days at L = 4.0, 100 days at L = 4.5 and 14 days at L = 5.0. The lower energy protons which were enhanced reach steady state conditions faster due to their greater losses. In Figure 5 the calculated change in the spectral parameter E_0 for different L-values are shown vs. time. One can see how rapid E_0 changes and stabilizes at high L-values. The E_0 values obtained after diffusion and loss processes have been working for 720 days have the $1/L^3$ dependence expected from a process conserving the two first adiabatic invariants and violating the third.

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Table 1

Set	$k \cdot 10^{-9}$	m	p	RMS
B	(2.4 ± 0.1)	0.0	10	0.86
C	0.73 ± 0.5	-0.035 ± 0.7	10.94 ± 0.2	0.82

Set B gives the value of the parameter k in Equation (5) giving the best fit to the data with m and p fixed. In set C the values of the parameters k, m, and p which give the best fit to the data allowing all three to be varied are shown. The RMS of the fit is given in decibels.

FIGURE CAPTIONS

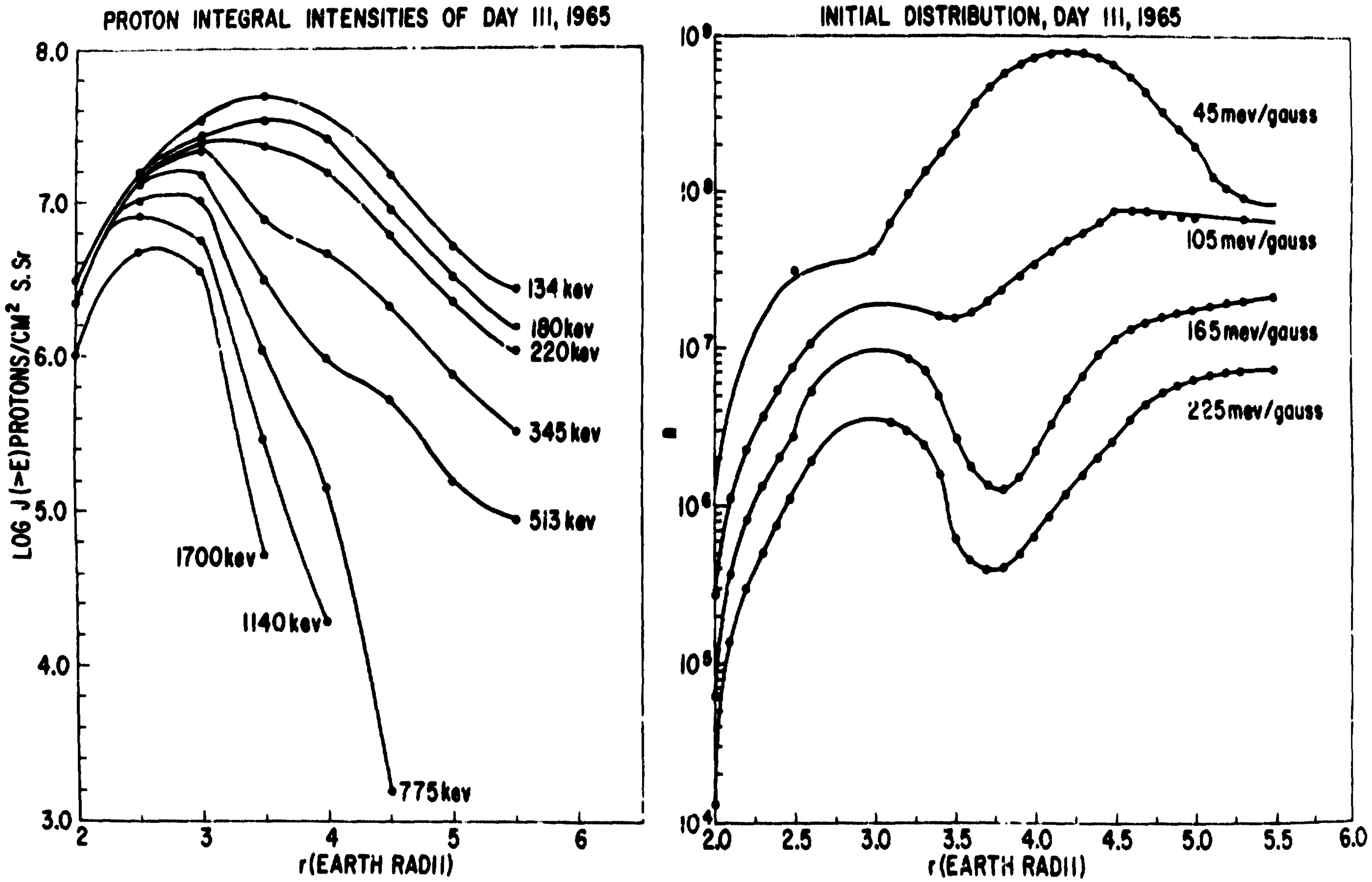
Figure 1 - The initial distribution. The left-hand side of the figure shows the integral proton fluxes above various energies versus radial distance as measured after the April 18 storm on day 111 of 1965. The right-hand side of the figure shows the distribution function n for different values of the magnetic moment plotted vs. radial distance.

Figure 2 - The time-behavior of the integral proton intensities at different L-values computed from the transport equation with two values of the diffusion coefficient, are compared with the experimentally measured values at $L = 3.0, 3.5, 4.0$ and 4.5.

Figure 3 - The left-hand side of the figure shows the measured distribution function n, for constant magnetic moments plotted vs. radial distance for 0, 20 and 36 days after the storm. The right-hand side of the figure shows for the same days after the storm the distribution function n as calculated from the diffusion equation (6) using $D_2 = 2.40 \cdot 10^{-9} r^{10}$ (earth radii)^2/day plotted for constant magnetic moments vs. radial distance.

Figure 4 - The time-evolution of the distribution function n calculated from the diffusion equation (6) using $D_2 = 2.40 \cdot 10^{-9} r^{10}$ (earth radii)^2/day plotted for constant magnetic moment vs. radial distance. The curves shown are for diffusion times of 0, 50, 150, 300 and 500 days.

Figure 5 - The spectral e-folding energy E_0 at L-values 2.5, 3.0, 3.5, 4.0, 4.5 and 5.0 computed from the solution of the transport equation and plotted vs. time.



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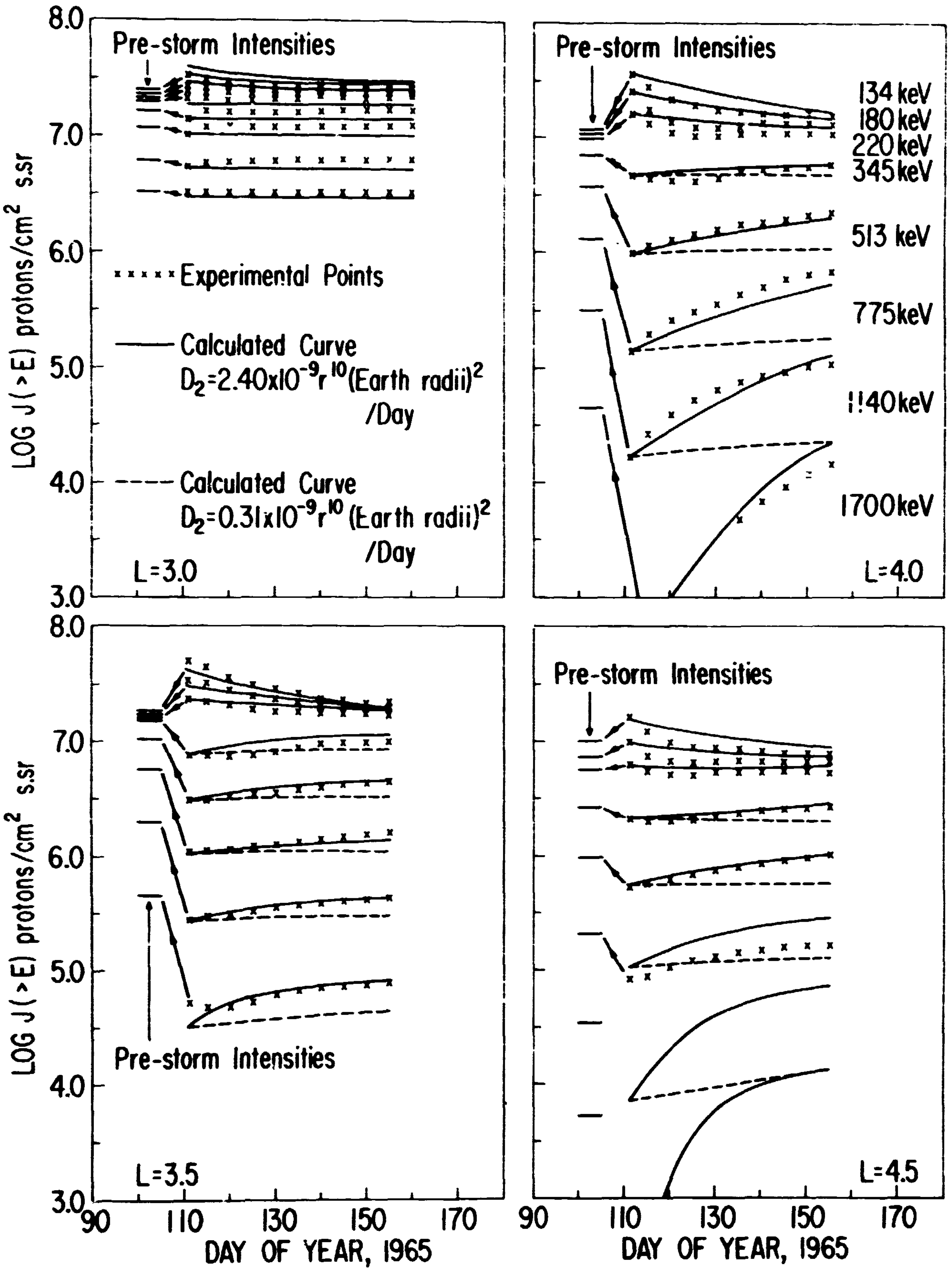

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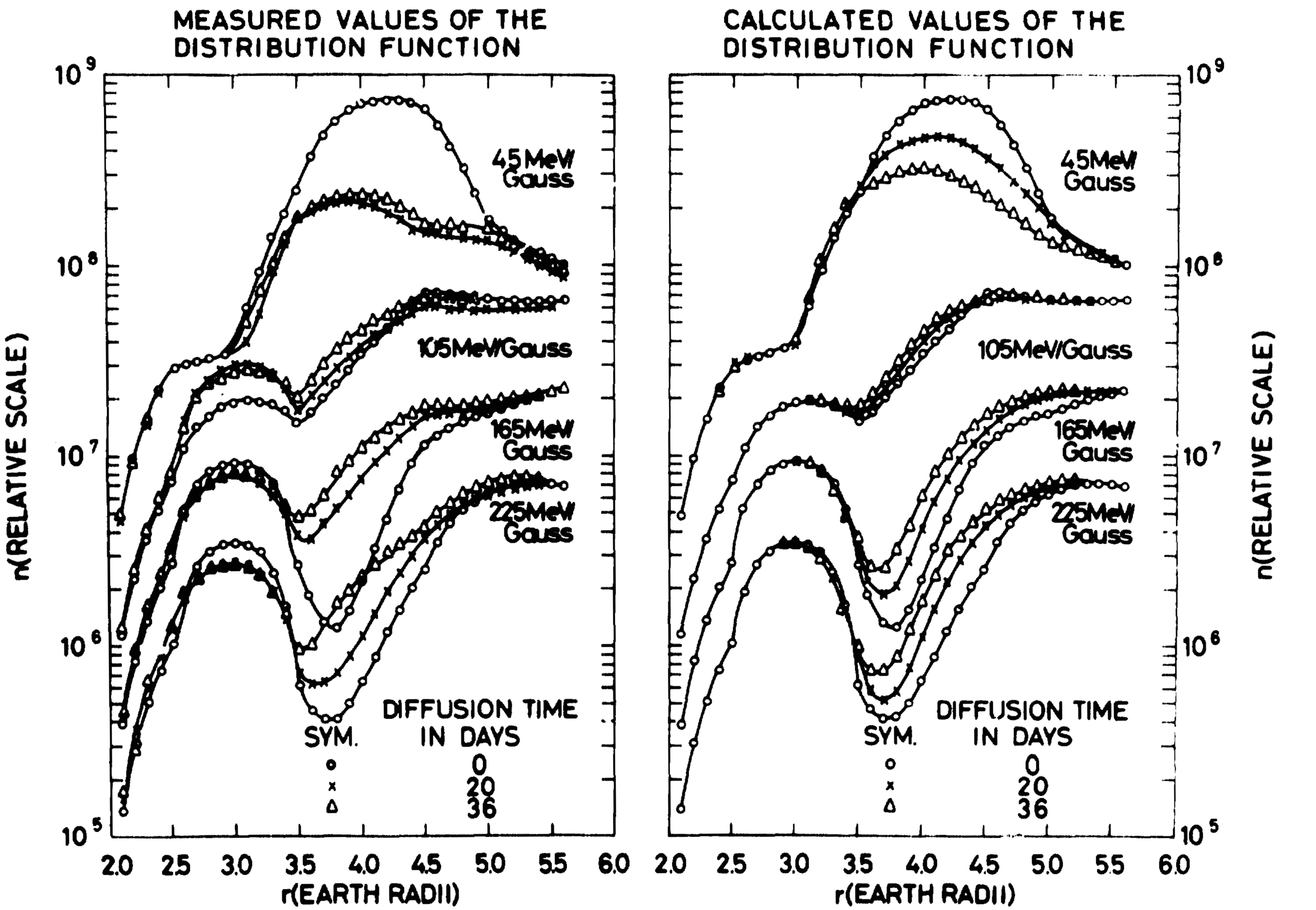

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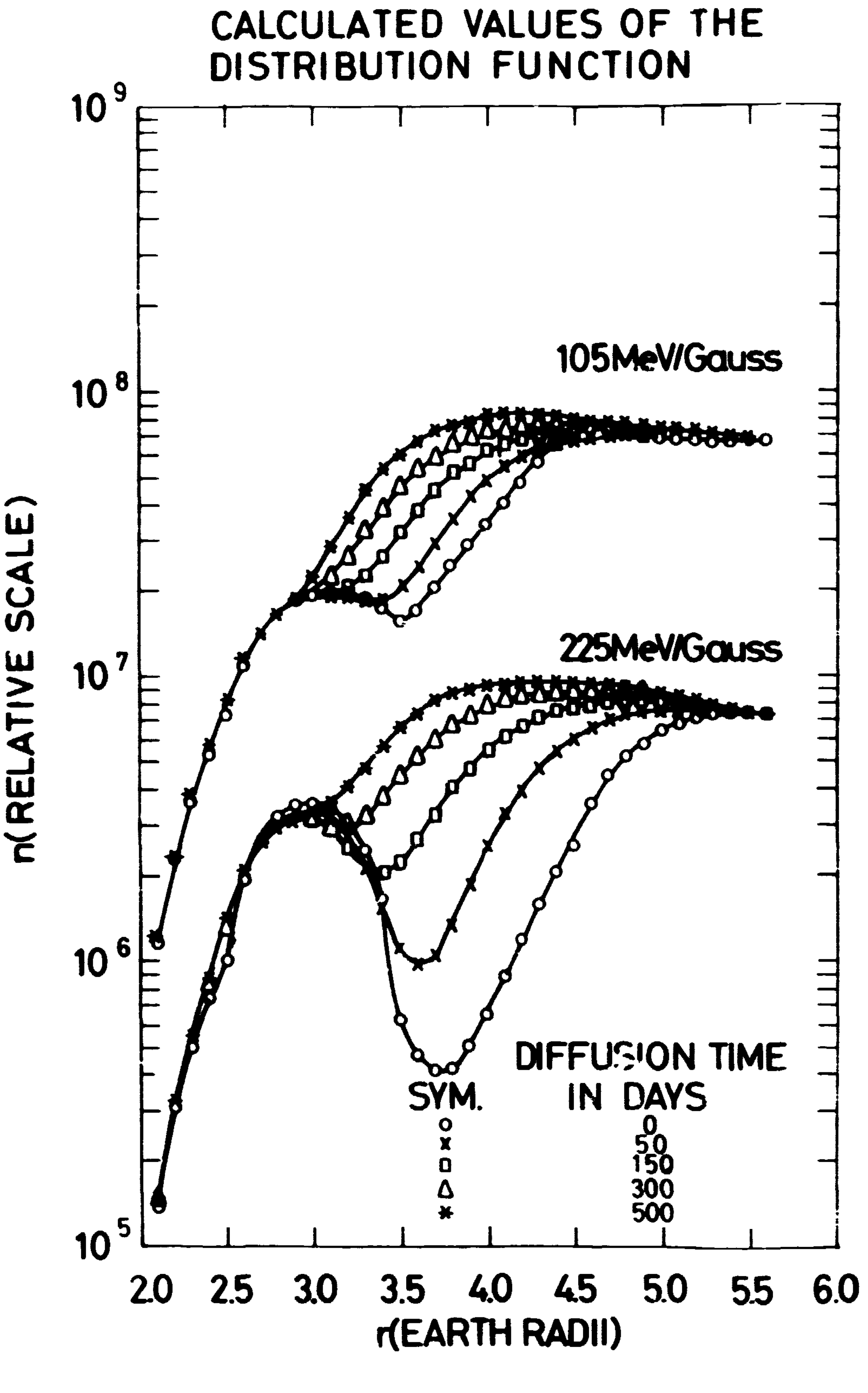

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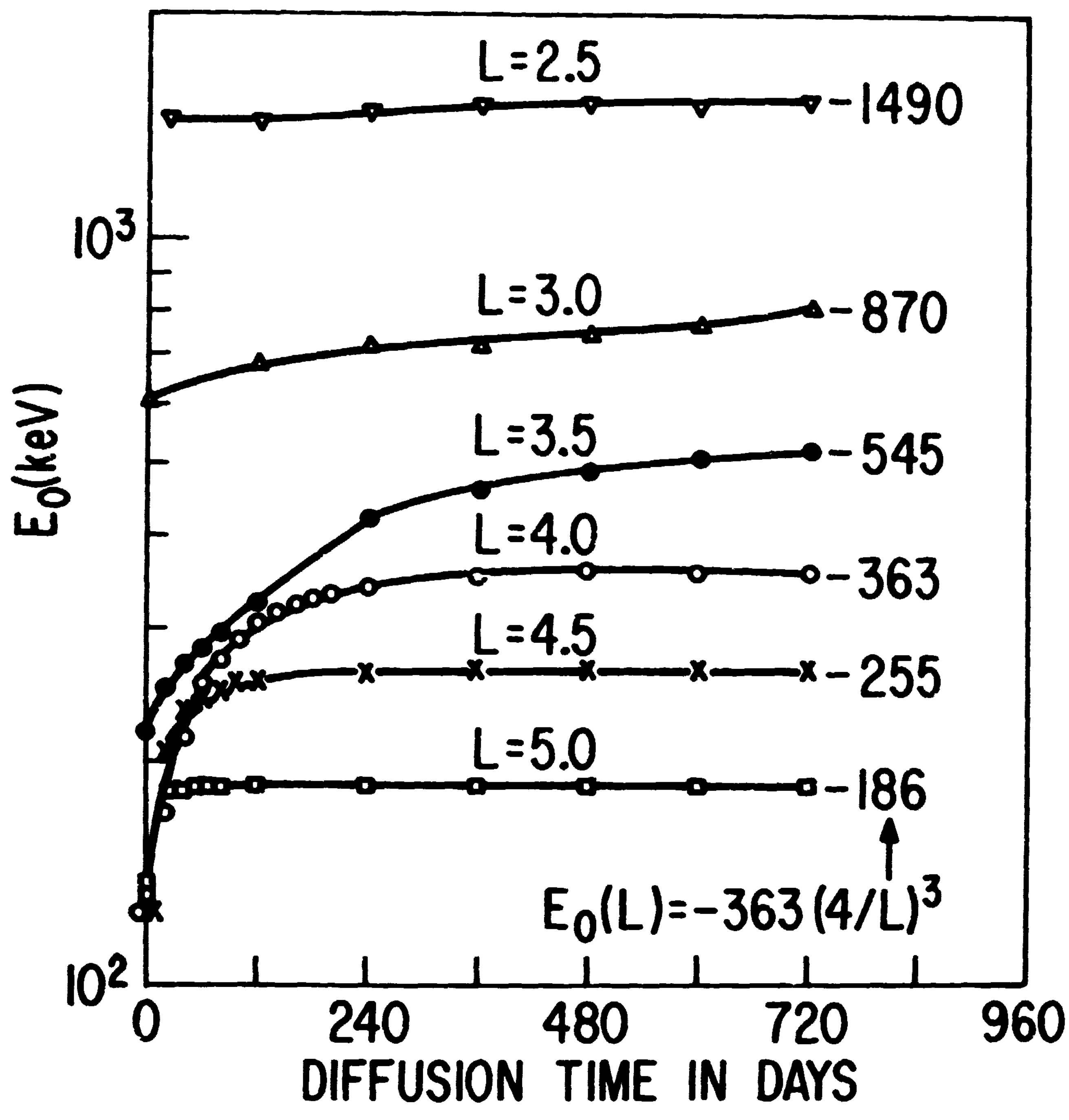

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